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ABSTRACT

The energy cost for ion production has been calculated for optically thin atomic helium, argon, and cesium gases by comparing the relative probabilities for the competing inelastic processes of excitation and ionization. Results were obtained for two cases: (1) a monoenergetic electron beam incident upon a cold neutral gas, and (2) the interaction of a thermal electron gas with cold neutrals. Experimental excitation cross sections were used in the helium calculations. The semiclassical Cryzinski method was used to determine theoretically the cross sections needed for the argon and cesium calculations. Results obtained by using theoretically determined helium excitation cross sections are compared with those obtained by using the experimental cross sections. The results are presented graphically in plots of ion-production cost (ev/ion) and ion-production rate versus electron energy. In general, this cost decreases smoothly with increasing electron energy in the thermal case and decreases irregularly with increasing electron energy in the beam case. A cursory analysis of the interaction of an electron beam with a plasma has been made from the viewpoint of determining the plasma conditions under which the results of each of the above-mentioned cases may be applied.

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INTRODUCTION

In plasma production and heating devices, it is advantageous to know the net energy cost for each ion produced, the ion-production rate, and the product of these two terms, which is the net power consumed in ion production. These quantities are needed to perform calculations on power balance and species continuity.

Since there are many possible methods of producing and maintaining plasmas, as well as a wide range of operating pressures and energies, it is obvious that one treatment cannot completely describe the ion-production parameters or costs for all plasma devices. Two plasma-production mechanisms are considered herein: (I) a monoenergetic electron beam incident upon a neutral gas, and (II) a low-pressure discharge in which the energy is added directly to the electrons. In both cases, the electron energy is transferred to the other species by collisions. These collisions may be either elastic or inelastic, the former resulting in gas heating and the latter in excitation or ionization processes. The energy transferred from the electrons is expended usefully if the atom is ionized but is lost if the target atom is excited and radiates away the excitation energy.

If the cross sections for the various inelastic processes are known, the ion-production cost may be determined by comparing the cross sections for the competing processes of excitation and ionization. Such cross sections have been experimentally determined for a few gases but, in general, experimental values of the inelastic excitation cross sections for gases of interest in plasma-production devices are not available and calculations of the ion-production costs have not been made. The good agreement obtained between the theoretical cross sections calculated by using the semiclassical Gryzinski method (ref. 1) and those experimental cross sections that are available indicates, however, that a fairly good approximation of the ion-production cost can be obtained by using these theoretical cross sections.

The purposes of this study are to calculate (1) the ion-production cost for case I, a monoenergetic electron beam incident upon neutral helium, argon, and cesium gases, and (2) the volume-ion-production cost, the ion-production rate, and the power consumed in maintaining a steady-state plasma for case II. In the latter case, the plasmas considered are partially ionized, optically thin helium, argon, and cesium plasmas, where a Maxwellian distribution of free-electron energies is assumed. The theoretical Gryzinski cross sections are used in both approaches.

THEORY

Assumptions and Limitations

The ion-production cost is calculated by assuming that the only important losses of free-electron energy occur by inelastic collisions, that is, ionization and excitation of bound electrons. In the ranges of electron energy or temperature of interest (from 2.5 to 50 ev), the energy loss in elastic collision is clearly negligible since the average energy loss per encounter is small. Furthermore, all electron-atom collisions are assumed to occur with ground-state atoms. The results presented are applicable only to low-pressure ($N_o \simeq 10^{12}$ to 10^{13}, $N_e \simeq 10^{11}$ to 10^{12}), optically thin plasmas for which cumulative inelastic impacts are improbable (ref. 2) due to the short lifetimes of the excited states.

The processes by which charged particles recombine may be considered separately in the ultimate power-balance calculation. The low-pressure discharges are primarily wall-controlled, and for wall recombination or radiative recombination, the energy is considered as being lost from the plasma. Although there may be an energy feedback to the electrons in three-body recombination, this process is not the dominant recombination process for the number densities and electron energies considered in this treatment.

Collisions between electrons and atomic ions may be important in the ion-production-cost calculations. Contributions of the effects of including the ionization and excitation of Ar^+ and Cs^+ were therefore included in this treatment, since these are the only ions of importance in the calculations.

In the analysis of the interaction of a monoenergetic electron beam with a partially ionized plasma, the possibility of generating plasma oscillations must be considered in addition to the collisional processes. These possibilities are included in the discussion of this interaction given in appendix A.

Development of Equations

Monoenergetic-beam case. - In this section, the equations necessary for determining the energy cost per ion will be developed for a monoenergetic electron beam of energy E_B incident upon a cold neutral gas. Using the symbols defined in appendix B, let the normalized probability for exciting the j^{th} level be defined as

$$P_j(E_B) = \frac{Q_j(E_B)}{Q_{tot}(E_B)} \qquad (1)$$

where $Q_j(E_B)$ is the cross section for exciting the j^{th} atomic energy level by impact with an electron of kinetic energy E_B, and $Q_{tot}(E_B)$ is the total inelastic cross section for the same electron. Similarly, the normalized probability for ionization by an electron with energy E_B is

$$P_1^+(E_B) = \frac{Q_{ion}(E_B)}{Q_{tot}(E_B)} \tag{2}$$

For kinetic energies of the incident electron such that $E_B - E_j \leq E^+$, where E_j and E^+ are the energies of the j^{th} excited state and the ionization energy, respectively, the energy cost for each ion formed by the beam is simply

$$\varphi_B = \frac{E_B}{P_1^+(E_B)} \qquad \text{ev/ion} \tag{3}$$

with the assumption that any excited state produced will lose the excitation energy by radiation

The calculation becomes more complex, however, if the electron energy, after the first inelastic collision $E_B - E_j$, is greater than the ionization energy, since this electron can still ionize another atom. When the beam energy is sufficiently high to cause a maximum of two ionizations, the expression for φ_B becomes

$$\varphi_B = \frac{E_B}{P_2^+(E_B)} \qquad \text{ev/ion} \tag{4}$$

where

$$P_2^+(E_B) = P_1^+(E_B) + \sum_j P_j(E_B) P_1^+(E_B - E_j) \tag{5}$$

When a maximum of three ionizations is possible, the equation for φ_B is

$$\varphi_B = \frac{E_B}{P_3^+(E_B)} \qquad \text{ev/ion} \tag{6}$$

where

$$P_3^+(E_B) = P_2^+(E_B) + \sum_i \sum_j P_j(E_B) P_i(E_B - E_j) P_1^+(E_B - E_j - E_i) \qquad (7)$$

Similar equations may be obtained for higher beam energies by including additional terms in the fashion of equations (4) to (7).

Electron-energy-distribution case. - In this section, the interaction between neutral gas atoms and an electron gas with a Maxwellian distribution of electron energies is analyzed. The theory is developed in the same manner as presented by Sovie and Klein (ref. 2).

The number of j^{th} excited states produced per unit volume per second by monoenergetic electrons of speed V_e is

$$\dot{N}_j = N_o N_e Q_j(V_e) V_e \qquad (cm^{-3})(sec^{-1}) \qquad (8)$$

where the cross sections employed in this section are expressed as a function of electron speed V_e. If there is a distribution of free-electron energies, equation (8) becomes

$$\dot{N}_j = N_o N_e \langle Q_j(V_e) V_e \rangle \qquad (cm^{-3})(sec^{-1}) \qquad (9)$$

where the brackets indicate an average value over the distribution function. The total energy expended in excitation processes per unit volume per second is, therefore,

$$\dot{E}_{j,tot} = \sum_j \dot{N}_j E_j = N_o N_e \sum_j \langle Q_j(V_e) V_e \rangle E_j \qquad ev/(cm^3)(sec) \qquad (10)$$

Similarly, the number of ions produced per cubic centimeter per second is

$$\dot{N}_{ion} = N_o N_e \langle Q_{ion}(V_e) V_e \rangle \qquad (cm^{-3})(sec^{-1}) \qquad (11)$$

and the energy expended in ionization processes is given by

$$\dot{E}_{ion} = N_o N_e \langle Q_{ion}(V_e) V_e \rangle E^+ \qquad ev/(cm^3)(sec) \qquad (12)$$

The net energy cost for producing singly ionized atoms (i.e., the volume-ion-production cost) is, therefore,

$$\varphi_T = \frac{\dot{E}_{ion} + \dot{E}_{j,tot}}{\dot{N}_{ion}} \qquad \text{ev/ion} \tag{13}$$

or

$$\varphi_T = \frac{E^+ + \sum_j \langle Q_i(V_e)V_e\rangle E_i}{\langle Q_{ion}(V_e)V_e\rangle} \qquad \text{ev/ion} \tag{14}$$

The net power consumed in ion production is the product of the ion-production rate and the energy cost per ion produced. In equation form,

$$W = 1.602\times10^{-19}\ N_o N_e \langle Q_{ion}(V_e)V_e\rangle \varphi_T \qquad \text{w/cm}^3 \tag{15}$$

When inelastic electron-ion interactions are included in the treatment, the expression for φ_T becomes

$$\varphi_T = \frac{\sum_i \langle Q_i(V_e)V_e\rangle E_i + \frac{N_{ion}}{N_o}\sum_k \langle Q_k'(V_e)V_e\rangle E_k}{\langle Q_{ion}(V_e)V_e\rangle + \frac{N_{ion}}{N_o}\langle Q_{ion}'(V_e)V_e\rangle} \qquad \text{ev/ion} \tag{16}$$

where i and k represent a summation over all inelastic processes, including ionization of the atom and singly ionized species, respectively, and Q_{ion}' is the ionization cross section for the singly charged species.

Calculation of Monoenergetic and Maxwellian Averaged Cross Sections

Cross sections. - The cross sections used in the helium calculations were those previously employed in reference 2, where the results of a number of individual investigations of helium excitation cross sections were combined to yield a credible self-consistent set of helium excitation functions. These excitation functions were represented by empirical equations describing the cross section as a function of electron velocity, multiplied by the electron velocity and averaged over a Maxwellian distribution of electron velocities to obtain the quantities $\langle Q(V_e)V_e \rangle$ mentioned in the preceding section. The argon and cesium cross sections used in the present treatment were calculated by using the semiclassical Gryzinski method (ref. 1). The total excitation cross section for helium was also calculated by this method for comparison with the results in reference 2.

The Cryzinski cross section expressions were derived in reference 1 in a classical manner with the assumption that the inelastic electron-atom collision occurs directly between the incident electron and a bound orbital electron. The kinetic energy of the bound electron is considered explicitly in this treatment, and consequently, the derived cross sections are generally more exact than those given by the classical Bohr-Thomson expression obtained for an atomic electron at rest with respect to the incident electron.

According to the Gryzinski formulation, the cross section for an inelastic electron-atom collision with an energy loss equal to or greater than U is given by

$$Q(U) = \frac{M\sigma_o}{U^2} g_j(E_2/U, E_1/U) \tag{17}$$

where

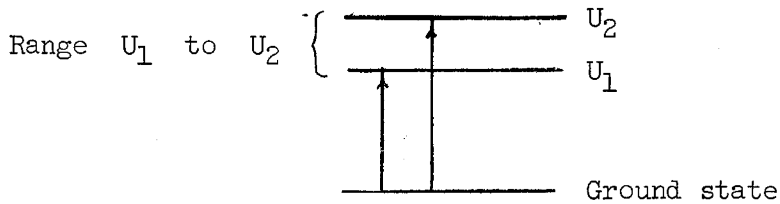
$$g_j(E_2/U, E_1/U) = \left\{\begin{array}{ll} \left(\dfrac{E_2}{E_1+E_2}\right)^{3/2} \dfrac{2}{3}\dfrac{E_1}{E_2} + \dfrac{U}{E_2}\left(1-\dfrac{E_1}{E_2}\right) - \left(\dfrac{U}{E_2}\right)^2 & \text{if } U + E_1 \leq E_2 \\ \left(\dfrac{E_2}{E_1+E_2}\right)^{3/2} \dfrac{2}{3}\left[\dfrac{E_1}{E_2} + \dfrac{U}{E_2}\left(1-\dfrac{E_1}{E_2}\right) - \left(\dfrac{U}{E_2}\right)^2\right]\left[\left(1+\dfrac{U}{E_1}\right)\left(1-\dfrac{U}{E_2}\right)\right]^{1/2} & \text{if } U + E_1 \geq E_2 \end{array}\right. \tag{18}$$

E_1 is the kinetic energy of the bound electron, and E_2 is the energy of the incident electron.

The symbol M denotes the number of equivalent electrons (same principal and azimuthal quantum number) in the outer shell of the target atom and serves as an effective probability factor that accounts for the number of electrons available for collisions. For an atom with a single outer-shell electron, the kinetic energy of the bound electron E_1 is equal to the ionization potential. For other atoms, however, E_1 is assumed to be equal to the sum of the total energy needed to remove all of the electrons in the outer shell divided by the number of outer-shell electrons. The validity of this assumption is shown in reference 3. Equation (17) is actually a definition of the ionization cross section when $U = E^+$. In this case, both electrons are free after an ionizing collision, and there is no limit on the energy transferred to the bound electron. The expression for the ionization cross section is then

$$Q_{ion} = \frac{M\sigma_o}{(E^+)^2} g_j(E_2/E^+, E_1/E^+) \tag{19}$$

In the calculation of excitation cross sections for bound electrons, the arrangement of the energy levels in the atom must be considered. The inelastic excitation cross section for an electron-induced transition between the ground state and an excited state is defined as the cross section for an energy loss restricted to the energy range U_1 to U_2 (see sketch (a)).



(a)

The symbol U_1 represents the energy difference between the state to be excited and the ground state, and U_2 represents the energy difference between the next higher excited state and the ground state. The incident electron abruptly "sees" the upper state once it has enough energy to excite it, and, consequently, that level serves as a semiclassical limit to the cross section function. Using the formulation of equation (17) gives the expression for the excitation cross section of a level at energy U_1 above the ground state simply as

$$Q_{exc} = Q(U_1) - Q(U_2) \tag{20}$$

The mean product of the theoretical cross section and the electron velocity integrated over a Maxwellian free-electron velocity distribution at kinetic temperature kT_e is

$$\langle Q(V_e)V_e \rangle = \frac{8\pi}{C_o} \int_{U_1}^{U_2} Q(E_2)\exp(-E_2/kT_e)E_2 \, dE_2 \tag{21}$$

where C_o is a normalization factor equal to $(2\pi kT_e)^{3/2}(m_e)^{1/2}$. The appropriate Q_j or Q_{ion} expressions from equation (18) are employed depending on the value of E_2 compared with $E_1 + U$. The limits of equation (21) refer to the excitation process. The upper limit for ionization would be infinity.

Atomic model for calculations. - The total inelastic excitation cross sections for ground-state atoms have been calculated by using equation (20) with U_1 equal to the first excitation potential and U_2 equal to the ionization potential. This approximation of the total inelastic cross section has been used in the thermal-electron-gas calculations.

The energy expended in each excitation E_m is taken as the mean energy between the first excited state and the ionization potential. This is a reasonable approximation in view of the inverse-square dependence of the excitation cross section on the energy and the high density of excited states near the continuum. In the monoenergetic-beam calculations for argon and cesium, the excitation cross sections for certain discrete levels were determined by using equation (20). In the region of highly excited electronic states, the energy difference between excited states is small, and the cross sections for the remaining levels can be very well approximated as those for one level, in a manner similar to that used in the total-excitation cross section calculation.

The energy levels used in the helium calculations for both cases are shown in figure 1. The energy levels used for the argon and cesium monoenergetic beam calculations are shown in figures 2 and 3, respectively. For two levels with nearly identical energies (e.g., the 6p $^2P^o_{1/2,3/2}$ states in cesium), the cross section is best approximated by considering the two levels as one with an energy equal to the mean energy of the two, and the next higher level serves as a limit to the cross section. Sheldon and Dugan (ref. 4) have used this procedure and obtained good agreement with experiment.

In the calculation of the cross sections for the cesium atom, which has one electron in the 6s orbital, the value for E_1 is the ionization potential, 3.89 electron volts, and $M = 1$. For the argon atom, E_1 is determined by takine one-sixth of the sum of the ionization potentials of the six equivalent 3p electrons (therefore, $M = 6$), which gives an average kinetic energy E_1 of 51.72 electron volts, while the iolization potential is 15.75 electron volts. For the helium atom, with two 1s electrons, $M = 2$, $E = 39.4$ electron volts, and the ionization potential is 24.56 electron volts.

The ionization of singly charged species may be important in the cesium and argon calculations for high electron kinetic temperatures and percentage ionization. The ionization potential of Cs^+ is 25.1 electron volts and that of Ar^+ is 27.6 electron volts. The value of E_1 for Cs^+ was approximated by extrapolating and averaging the ionization potentials of the six electrons in the 5p subshell and was estimated as 55 electron volts. The value of E_1 for the Ar^+ calculations is one-fifth of the sum of the ionization potentials of the five 3p electrons and is equal to 58.5 electron volts.

RESULTS

The ion-production costs for the monoenergetic-beam case using the energy levels shown in figures 1 to 3 are presented as a function of beam energy by the solid curves in figure 4. For the three gases, this cost drops very sharply as the beam energy increases from the ionization potential to a few volts above the ionization potential. The costs for the helium and argon atoms are seen to decrease in a slightly irregular manner as the beam energy is further increased. The relative magnitudes of the curves are as one would expect from an inspection of the atomic structure of the atoms involved. The cost is highest for helium (highest ionization energy and first excited state) and lowest for cesium (lowest ionization potential and first excited state). Due to the large number of terms carried in the determination of φ_B (eqs. (4) to (7)), the calculations had to be terminated at an energy that would allow a maximum of three ionizations. Consequently, comparison of the costs for the three atoms cannot be made over a wide range of energies. The dotted portion of the cesium curve was obtained by considering only two cross sections, the total excitation cross section with an associated energy loss of 2.7 electron volts and the ionization cross section. This approximation allowed the cesium results to be carried to higher beam energies, although the accuracy of these results is unknown.

The results of the electron energy distribution calculations are shown in figure 5, which is a plot of volume-ion-production cost versus electron kinetic temperature. The volume-ion-production cost for each of the atoms is seen to decrease sharply with increasing temperature up to about 12 electron volts and then to decrease slowly as the temperature is increased to 50 electron volts. The results obtained by using the Gryzinski approximation for the total-excitation cross section for helium are seen to agree favorably with the results of reference 2. As would be expected, the energy cost per ion was greatest for helium and least for cesium.

In order to facilitate the use of these results in other calculations, such as a power balance, two additional curves are presented for each of the gases. An ion-production-rate parameter $\dot{N}_{ion}/N_o N_e$ (eq. (11)) is plotted as a function of electron kinetic temperature in figure 6. Figure 7 shows the variation with electron kinetic temperature of the power consumed in ion production $\varphi_T \dot{N}_{ion}/N_o N_e$ (eq. (15)).

The effects of inelastic electron-ion collisions on the φ_T values presented in figure 5 are shown in figure 8 for a fraction ionized of 10 percent in argon and cesium. These collisions are not important up to about 40 electron volts for helium (ref. 2) and were therefore not considered. The results for the monoenergetic beam are compared with the thermal results of the average electron kinetic temperature in figure 9. The costs for the thermal case are seen to be considerably lower than those for the monoenergetic beam. This result was expected since the high-energy tail of the distribution operates in a region where ionization is more probable than excitation.

The results of the collision-time comparisons are shown in figure 10 for a neutral particle density of 10^{12} cm^{-3}, a fraction ionized of 10 percent ($N_e = 10^{11}$ cm^{-3}) and an electron kinetic temperature varying from 0.5 to 10 electron volts. The electron-electron energy relaxation time and the inelastic electron-neutral collision time are plotted versus the monoenergetic-electron-beam energy in figure 10.

The growth time for plasma oscillations (considering the two-stream instability analysis (ref. 5)) has been calculated by using the equation given in appendix A. For a plasma electron density of 10^{11} cm^{-3}, the growth time would be of the order of 10^{-9} second, which is very much less than the times shown in figure 10. Consequently, an electron beam impinging on the partially ionized plasma mentioned above would probably have its energy randomized to some energy different from the beam energy and the electron kinetic energy of the plasma. The net energy cost per ion in this case would likely be below that for a monoenergetic beam and approach the thermal results at the average electron kinetic energy shown in figure 9.

CONCLUDING REMARKS

Theoretical calculations of the total excitation cross sections for helium, argon, and cesium have been made by using the semiclassical Gryzinski method. These cross sections have been used in the calculation of the ion-production cost for (1) a monoenergetic electron beam incident upon a cold neutral gas, and (2) a thermal electron gas interacting with neutrals. The results obtained by using the Gryzinski approximation of the total excitation cross section for helium agree favorably with those obtained by using empirically obtained cross sections (ref. 2).

The energy cost per ion formed is higher for the monoenergetic beam than for the thermal electron gas at a kinetic temperature equal to the beam energy. The ion-production-cost curves exhibit the expected trends, namely, that the atoms with the higher ionization potential and first excited state have the higher cost per ion and that these costs decrease with increasing electron energy.

The results for the monoenergetic-beam analysis are applicable for a beam incident upon a cold neutral gas. If the gas is partially ionized, however, the beam energy will probably be randomized by setting up plasma oscillations, and the energy cost per ion in this case would be less than that calculated in the above case.

The electron-energy-distribution results apply to steady-state, partially ionized, tenous plasmas that are optically thin and in which a Maxwellian distribution of electron energies prevails. The results presented represent only a portion of the power-consumption rate or species-production rate in an actual experiment. There will be additional terms to account for recombination or wall losses, but these depend strongly on the experimental configurations. There will also be additional terms if the plasma is heated, accelerated, or does work.

APPENDIX A

INTERACTION OF MONOENERGETIC ELECTRON BEAM WITH PARTIALLY IONIZED PLASMA

An electron beam incident upon a partially ionized nonequilibrium plasma may interact in a number of different modes. The electron beam may transfer its energy directly to the plasma neutrals or electrons, but it may also initiate plasma oscillations or instabilities. If the beam interacts with the neutrals, it will ionize them directly, as in the monoenergetic-electron-beam case. If the electron beam interacts with the plasma electrons (which are assumed to have a Maxwellian distribution of electron energies), it will be thermalized with the free-electron gas. If plasma instabilities are generated, the electron beam energy may initially be randomized at some energy different from the beam energy and the plasma electron energy. A cursory analysis of an electron-beam - plasma interaction may be carried out simply by comparing (1) the time between electron-neutral collision τ_c, (2) the energy relaxation time for an electron-beam - thermal-electron-gas interaction, and (3) the growth time for plasma instabilities. The time between inelastic electron-neutral collision is given by the expression

$$\tau_c = \frac{1}{N_o Q_T(V_e) V_e} \tag{A1}$$

The energy relaxation time for electron-electron collisions is taken from Spitzer (ref. 6) and may be written as

$$\tau_E = \frac{V_B^3}{6.44\times10^{18} N_E \ln\Lambda\, G\left(\sqrt{\frac{E_B}{kT_e}}\right)} \tag{A2}$$

where V_B is the velocity of the beam electrons, $\Lambda = \frac{3}{2_e^3}\left(\frac{k^3 T_e^3}{\pi N_e}\right)^{1/2}$, and

$G(\sqrt{E_B/kT_e})$ is a function of the ratio of the beam energy to the free-electron energy, as defined in appendix B.

A qualitative analysis of the two-stream instability (ref. 5) indicates that the growth time for plasma oscillations is proportional to the plasma period, therefore,

$$\tau_g \propto \frac{1}{\omega_p} \tag{A3}$$

where ω_p is the plasma frequency equal to $5.64\times10^4(N_e)^{1/2}$. The energy relaxation time and the electron-neutral collision time are plotted in figure 10, but, in general, τ_g will be much less than either of these times.

APPENDIX B

SYMBOLS

C_o	noralization factor, $(2\pi kT_e)^{3/2} m_e^{1/2}$, $(ev^{3/2})(g^{1/2})$
E	energy, ev
E_m	mean energy between first excited state and ionization potential, ev
E_1	kinetic energy of bound electron in Gryzinski formulas, ev
E_2	energy of incident electron in Gryzinski formulas, ev
E^+	ionization energy, ev
$\dot{E}_{ion}$	energy expended in ionization processes, ev/(cm^3)(sec)
$\dot{E}_{j,tot}$	total energy expended in excitation processes, ev/(cm^3)(sec)
e	electronic charge, 4.8024×10^{-10} esu
$G(x)$	$\dfrac{\Phi(x) - x\dfrac{d\Phi(x)}{dx}}{2x^2}$, where $x = \left(\dfrac{E_B}{kT_e}\right)^{1/2}$ and $\Phi(x) = \dfrac{2}{\pi^{1/2}} \displaystyle\int_0^x d^{-y^2}\, dy$
kT_e	electron kinetic temperature, ev
M	number of equivalent electrons in outer shell of target atom in Gryzinski formulas
m_e	electron mass, 9.11×10^{-28} g
N	number density, cm^{-3}
$\dot{N}_{ion}$	ion-production rate, (cm^{-3})(sec^{-1})
$\dot{N}_j$	production rate of excited states, (cm^{-3})(sec^{-1})
P_j	normalized probability for excitation of jth atomic energy level
P_1^+	normalized probability for ionization
Q	inelastic cross section, cm^2
Q'_{ion}	ionization cross section for inelastic electron-ion interactions, cm^2

$Q(V_e)$	inelastic cross section represented as function of electron velocity V_e, cm^2
U	energy loss in Gryzinski formulas, ev
V	velocity, cm/sec
W	power consumed in ionization, w/cm^3
σ_o	constant in Gryzinski calculations, 6.53×10^{-14} $(cm^2)(ev^2)$
τ_c	electron-neutral collision time, sec
τ_E	energy relaxation time for electron-electron collisions, sec
τ_g	growth time for plasma instabilities, sec
φ_B	ion-production cost for monoenergetic beam, ev/ion
φ_T	volume-ion-production cost for electron-distribution case, ev/ion
ω_p	plasma frequency, sec^{-1}

Subscripts:

B	electron beam
e	electron
ion	ion
j	j^{th} excited state
o	neutral particle
tot	total - sum of all inelastic processes

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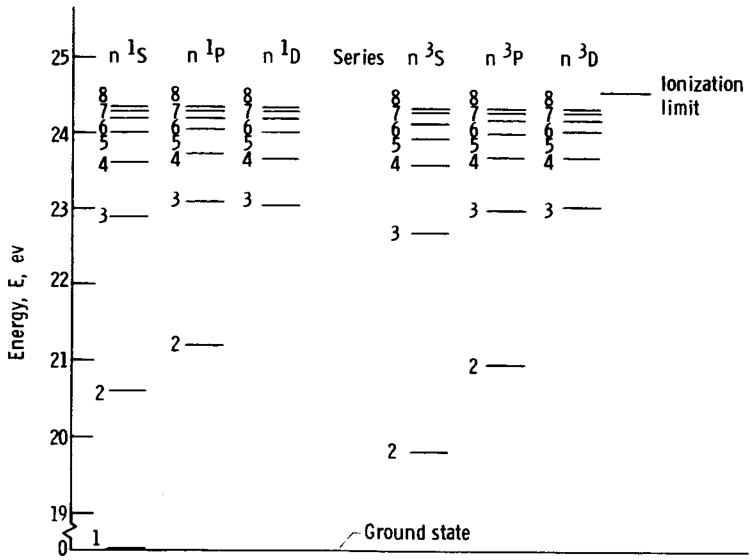

Figure 1. - Energy levels used in helium calculations.

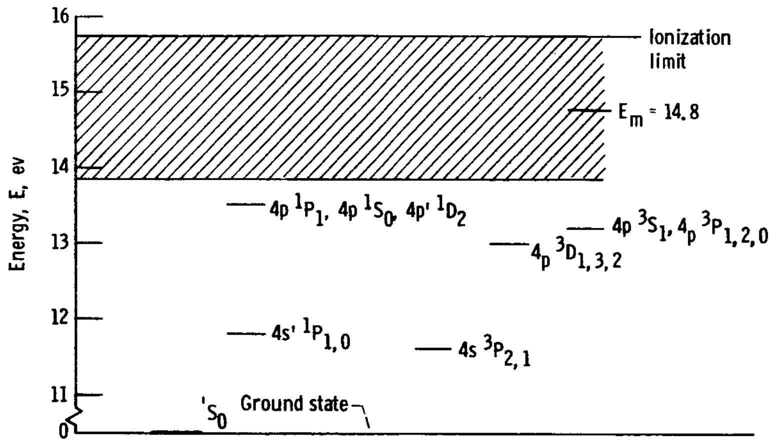

Figure 2. - Energy levels used in argon calculations.

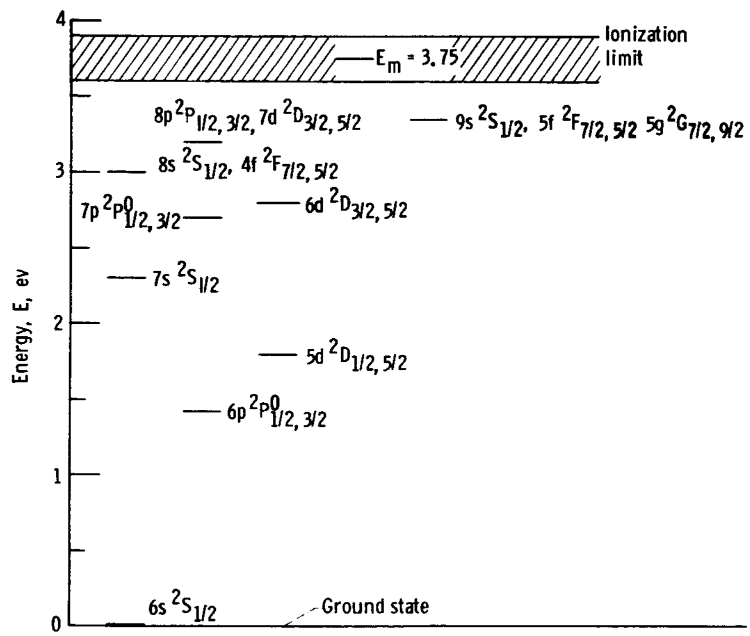

Figure 3. - Energy levels used in cesium calculations.

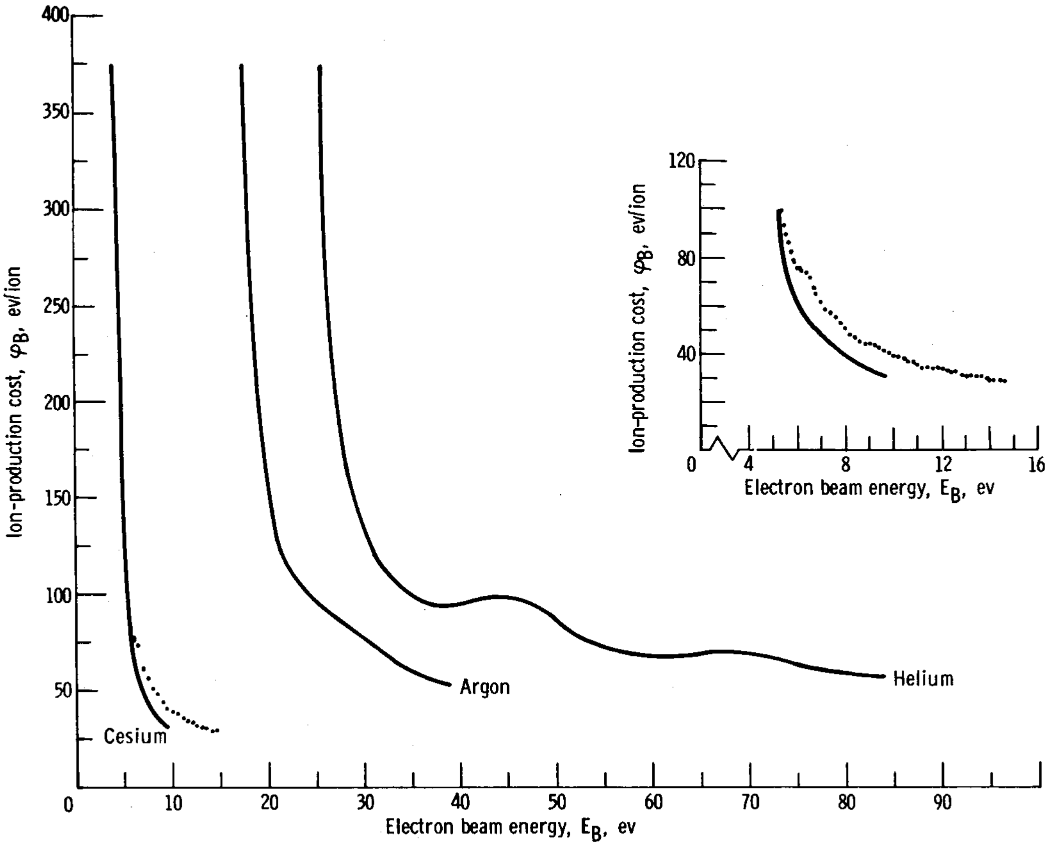

Figure 4. - Ion-production cost for monoenergetic-beam case.

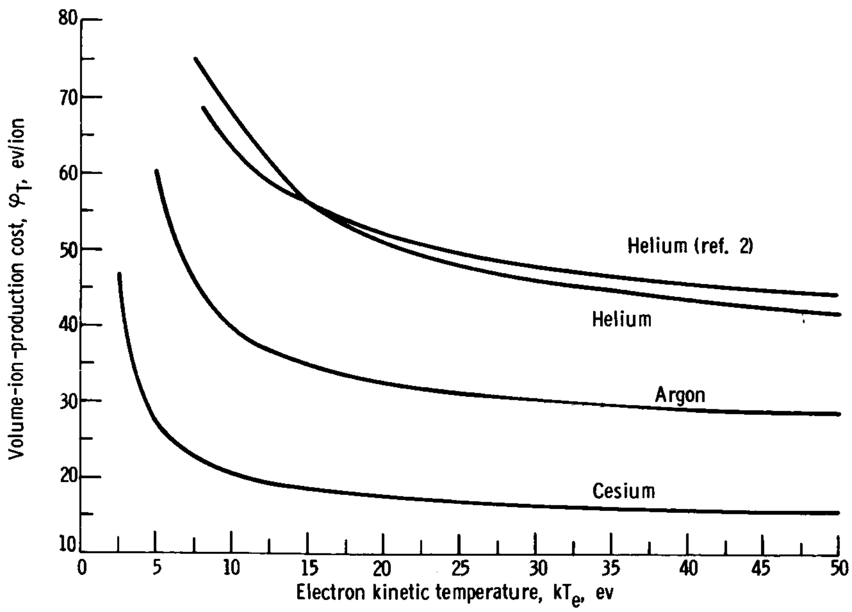

Figure 5. - Volume-ion-production cost as function of electron kinetic temperature.

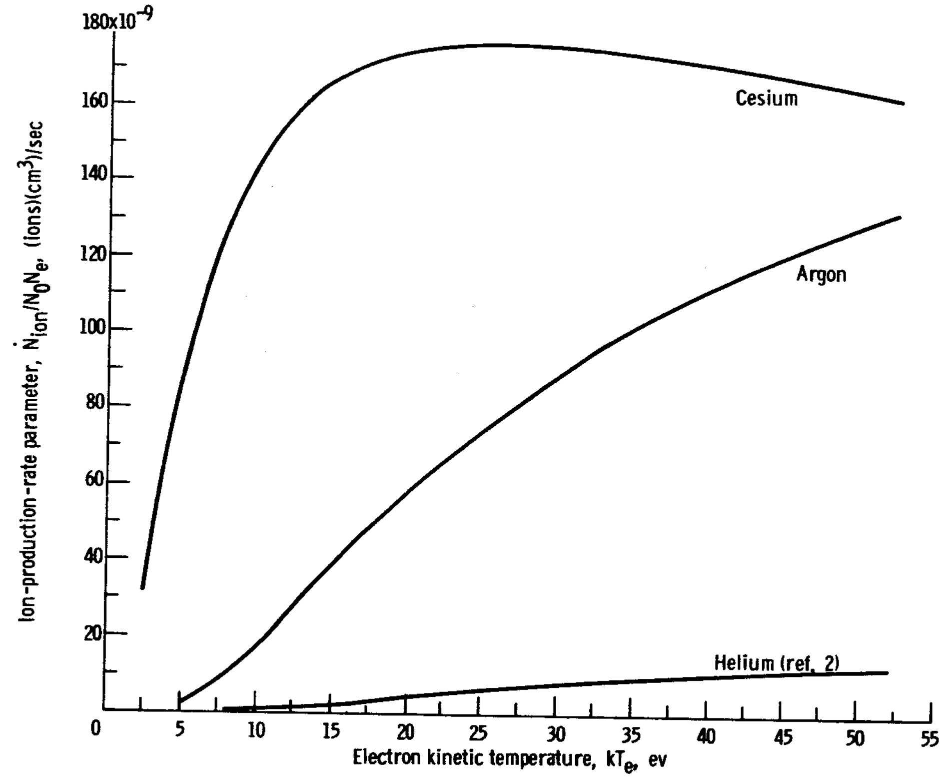

Figure 6. - Ion-production-rate parameter as function of electron kinetic temperature.

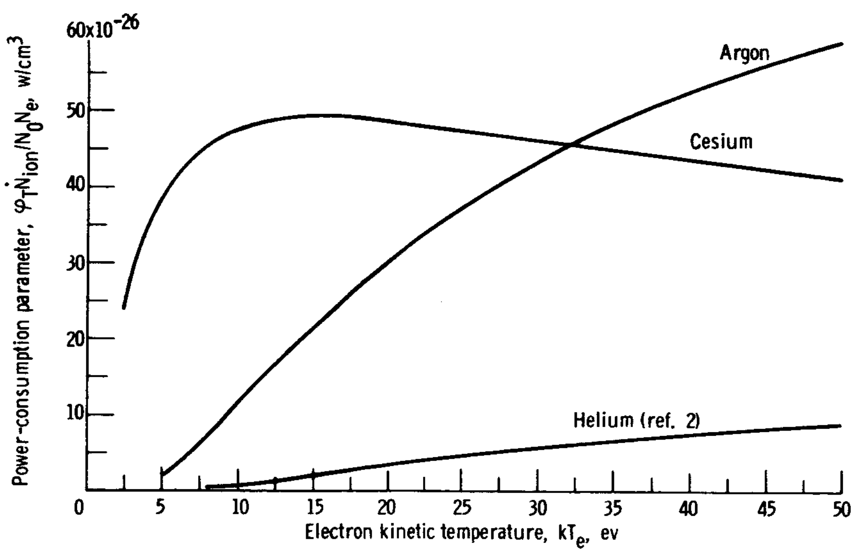

Figure 7. - Power-consumption parameter as function of electron kinetic temperature.

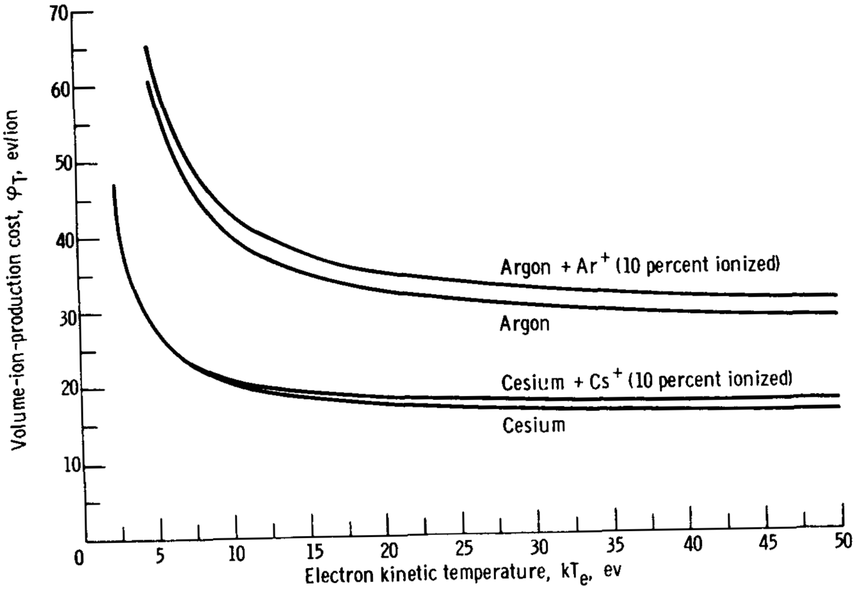

Figure 8. - Effect of electron-ion interactions on ion-production cost.

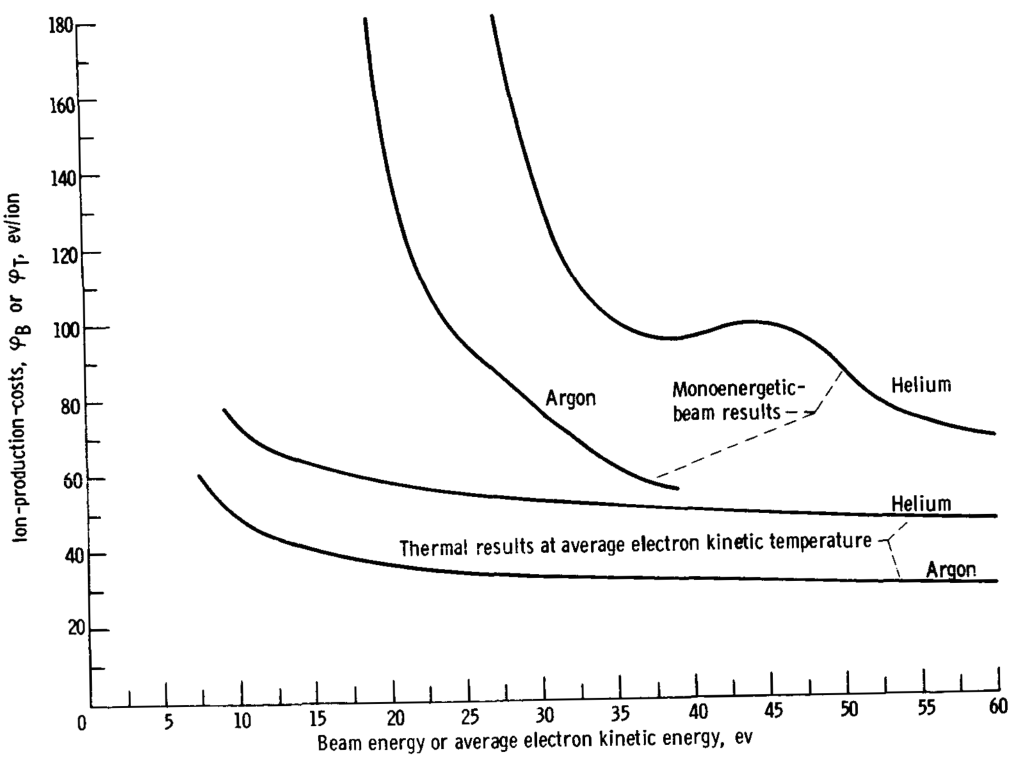

Figure 9. - Comparison of monoenergetic and thermal results at average electron kinetic temperature.

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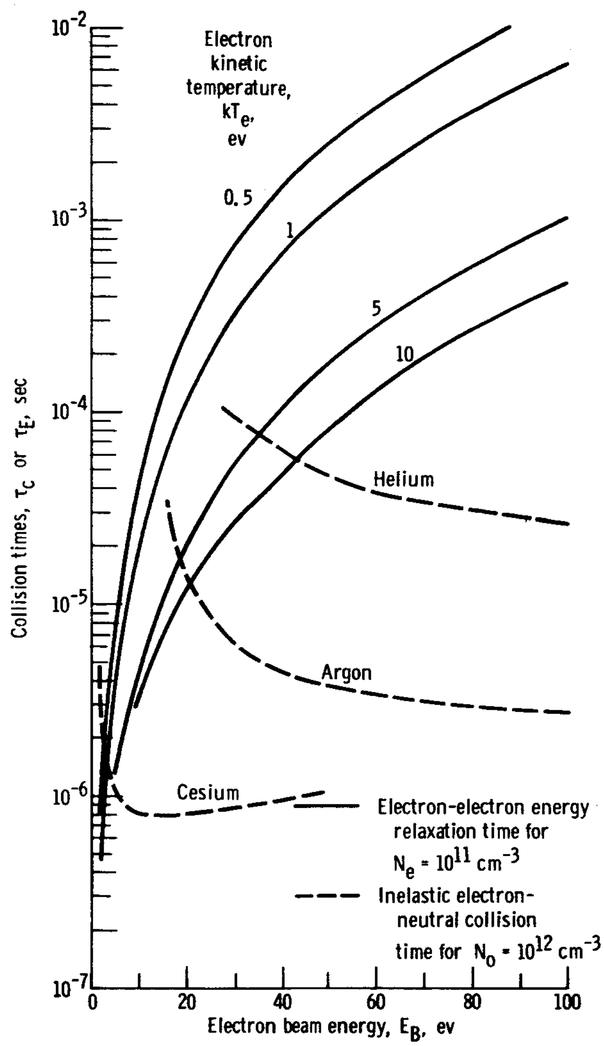

Figure 10. - Collision time comparison.